Eur. Phys. J. C 35, 259–265 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2004-01851-0

THE EUROPEAN
PHYSICAL JOURNAL C

$SU(3)_c \otimes SU(4)_\mathrm{L} \otimes U(1)_X$ model for three families

L.A. Sánchez[1], F.A. Pérez[1], W.A. Ponce[2]

[1] Escuela de Física, Universidad Nacional de Colombia, A.A. 3840, Medellín, Colombia
[2] Instituto de Física, Universidad de Antioquia, A.A. 1226, Medellín, Colombia

Received: 7 February 2004 /
Published online: 5 May 2004 –

Abstract. An extension of the standard model to the local gauge group $SU(3)_c \otimes SU(4)_\mathrm{L} \otimes U(1)_X$ as a three-family model is presented. The model does not contain exotic electric charges and we obtain a consistent mass spectrum by introducing an anomaly-free discrete Z_2 symmetry. The neutral currents coupled to all neutral vector bosons in the model are studied. By using experimental results from the CERN LEP, SLAC Linear Collider and atomic parity violation data we constrain the mixing angle between two of the neutral currents in the model and the mass of the additional neutral gauge bosons to be $-0.0032 \leq \sin\theta \leq 0.0031$ and $0.67\,\mathrm{TeV} \leq M_{Z_2} \leq 6.1\,\mathrm{TeV}$ at 95% C.L., respectively.

1 Introduction

The standard model (SM), based on the local gauge group $SU(3)_c \otimes SU(2)_\mathrm{L} \otimes U(1)_Y$ [1], can be extended in several different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitutes its simplest extension and has profound consequences, as the implementation of the see-saw mechanism, and the enlarging of the possible number of local abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one Higgs representation, and, third, by enlarging the local gauge group. In this last direction $SU(4)_\mathrm{L} \otimes U(1)_X$ as a flavor group has been considered before in the literature [2–4] which, among its best features, provides us with an alternative to the problem of the number N_f of families, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, N_c being the number of colors of $SU(3)_c$ (also known as QCD). Moreover, this gauge structure has been used recently in order to implement the so-called little Higgs mechanism [4].

The analysis of the local gauge structure $SU(3)_c \otimes SU(4)_\mathrm{L} \otimes U(1)_X$ (hereafter the 3-4-1 group) presented in the appendix of [3] shows that we may write the most general electric charge operator for this group as

$$Q = aT_{3\mathrm{L}} + \frac{b}{\sqrt{3}}T_{8\mathrm{L}} + \frac{c}{\sqrt{6}}T_{15\mathrm{L}} + XI_4, \tag{1}$$

where a, b and c are free parameters, $T_{i\mathrm{L}} = \lambda_{i\mathrm{L}}/2$, with $\lambda_{i\mathrm{L}}$ the Gell-Mann matrices for $SU(4)_\mathrm{L}$ normalized as $\mathrm{Tr}(\lambda_i\lambda_j) = 2\delta_{ij}$, and $I_4 = \mathrm{Dg}(1,1,1,1)$ is the diagonal 4×4 unit matrix. The X values are fixed by anomaly cancellation of the fermion content in the possible models and an eventual coefficient for XI_4 can be absorbed in the X hypercharge definition. The free parameters a, b and c fix the gauge boson structure of the electroweak sector $[SU(4)_\mathrm{L} \otimes U(1)_X]$, and also the electroweak charges of the scalar representations which are fully determined by the symmetry breaking pattern implemented. In particular $a = 1$ gives the usual isospin of the known electroweak interactions, with b and c remaining as free parameters, producing an infinite plethora of possible models.

Restricting the particle content of the model to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are generated [3]. In particular, the restriction to ordinary electric charges, in the fermion, scalar and gauge boson sectors, allows only for two different cases for the simultaneous values of the parameters b and c, namely: $b = c = 1$ and $b = 1, c = -2$, which become a convenient classification scheme for these types of models. Models in the first class differ from those in the second one not only in their fermion content but also in their gauge and scalar boson sectors. Four of the identified models without exotic electric charges are three-family models in the sense that anomalies cancel among the three families of quarks and leptons in a non-trivial fashion. Two of them are models for which $b = c = 1$, and one of them has been analyzed in [3]. The other two models belong to the class for which $b = 1, c = -2$ and one of them, the so-called "Model **E**" in the appendix of [3], will be studied in this paper. It is worth noticing that in the four different models at least one of the three families is treated differently.

This paper is organized as follows. In the next section we describe the fermion content of the particular model we are going to study. In Sect. 3 we introduce the scalar sector. In Sect. 4 we study the gauge boson sector, paying special attention to the neutral currents present in the model and their mixing. In Sect. 5 we analyze the fermion mass spectrum. In Sect. 6 we use experimental results in

Table 1. Anomaly-free fermion structure of Model **E** from [3]

$Q_{1\mathrm{L}} = \begin{pmatrix} d_1 \\ u_1 \\ U_1 \\ D_1 \end{pmatrix}_\mathrm{L}$	$d^c_{1\mathrm{L}}$	$u^c_{1\mathrm{L}}$	$U^c_{1\mathrm{L}}$	$D^c_{1\mathrm{L}}$
$[3, 4^*, \frac{1}{6}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$
$Q_{j\mathrm{L}} = \begin{pmatrix} u_j \\ d_j \\ D_j \\ U_j \end{pmatrix}_\mathrm{L}$	$u^c_{j\mathrm{L}}$	$d^c_{j\mathrm{L}}$	$D^c_{j\mathrm{L}}$	$U^c_{j\mathrm{L}}$
$[3, 4, \frac{1}{6}]$	$[3^*, 1, -\frac{2}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, \frac{1}{3}]$	$[3^*, 1, -\frac{2}{3}]$
$L_{\alpha\mathrm{L}} = \begin{pmatrix} e^-_\alpha \\ \nu_{e\alpha} \\ N^0_\alpha \\ E^-_\alpha \end{pmatrix}_\mathrm{L}$	$e^+_{\alpha\mathrm{L}}$	$E^+_{\alpha\mathrm{L}}$		
$[1, 4^*, -\frac{1}{2}]$	$[1, 1, 1]$	$[1, 1, 1]$		

order to constrain the mixing angle between two of the neutral currents in the model and the mass scale of the new neutral gauge bosons. In the last section we summarize the model and state our conclusions.

2 The fermion content of the model

In what follows we assume that the electroweak gauge group is $SU(4)_\mathrm{L} \otimes U(1)_X$ which contains $SU(2)_\mathrm{L} \otimes U(1)_Y$ as a subgroup. We will consider the case of a non-universal hypercharge X in the quark sector, which implies anomaly cancellation among the three families in a non-trivial fashion.

Here we are interested in studying the phenomenology of three-family models without exotic electric charges and with values $b = 1$, $c = -2$ for the parameters in the electric charge generator in (1). As an example we take Model **E** of [3] for which the electric charge operator is given by $Q = T_{3\mathrm{L}} + T_{8\mathrm{L}}/\sqrt{3} - 2T_{15\mathrm{L}}/\sqrt{6} + XI_4$. This model has the anomaly-free fermion structure as given in Table 1. where $j = 2, 3$ and $\alpha = 1, 2, 3$ are two- and three-family indexes, respectively. The numbers in parentheses refer to the $[SU(3)_C, SU(4)_\mathrm{L}, U(1)_X]$ quantum numbers, respectively. Notice that, if needed, the lepton structure of the model can be augmented with an undetermined number of neutral Weyl singlet states $N^0_{\mathrm{L},n} \sim [1, 1, 0]$, $n = 1, 2, \ldots$, without violating our assumptions, neither the anomaly constraint relations, because singlets with no X charges are as good as not being present as far as anomaly cancellation is concerned.

3 The scalar sector

Our aim is to break the symmetry, following the pattern

$$
\begin{aligned}
&SU(3)_c \otimes SU(4)_\mathrm{L} \otimes U(1)_X \\
&\to SU(3)_c \otimes SU(3)_\mathrm{L} \otimes U(1)_X \\
&\to SU(3)_c \otimes SU(2)_\mathrm{L} \otimes U(1)_Y \\
&\to SU(3)_c \otimes U(1)_Q,
\end{aligned}
$$

where $SU(3)_c \otimes SU(3)_\mathrm{L} \otimes U(1)_X$ refers to the so-called 3-3-1 structure introduced in [5]. At the same time we want to give masses to the fermion fields in the model. With this in mind we introduce the following four Higgs scalars: $\phi_1[1, 4^*, -1/2]$ with a vacuum expectation value (VEV) aligned in the direction $\langle\phi_1\rangle = (0, v, 0, 0)^\mathrm{T}$; $\phi_2[1, 4^*, -1/2]$ with a VEV aligned as $\langle\phi_2\rangle = (0, 0, V, 0)^\mathrm{T}$; $\phi_3[1, 4, -1/2]$ with a VEV aligned in the direction $\langle\phi_3\rangle = (v', 0, 0, 0)^\mathrm{T}$, and $\phi_4[1, 4, -1/2]$ with a VEV aligned as $\langle\phi_4\rangle = (0, 0, 0, V')^\mathrm{T}$, with the hierarchy $V \sim V' >> \sqrt{v^2 + v'^2} \simeq 174\,\mathrm{GeV}$ (the electroweak breaking scale).

4 The gauge boson sector

In the model there are a total of 24 gauge bosons: One gauge field B^μ associated with $U(1)_X$, the 8 gluon fields associated with $SU(3)_c$ which remain massless after breaking the symmetry, and another 15 gauge fields associated with $SU(4)_\mathrm{L}$ which, for $b = 1$ and $c = -2$, can be written as

$$
\frac{1}{2}\lambda_\alpha A^\mu_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} & X^{0\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} & X^{-\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 & Y^{-\mu} \\ \bar{X}^{0\mu} & X^{+\mu} & Y^{+\mu} & D^\mu_4 \end{pmatrix},
$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, $D^\mu_3 = -2A^\mu_8/\sqrt{6} + A^\mu_{15}/\sqrt{12}$, and $D^\mu_4 = -3A^\mu_{15}/\sqrt{12}$.

After breaking the symmetry with $\langle\phi_1\rangle + \langle\phi_2\rangle + \langle\phi_3\rangle + \langle\phi_4\rangle$ and using for the covariant derivative for 4-plets $\mathrm{i}D^\mu = \mathrm{i}\partial^\mu - g\lambda_\alpha A^\mu_\alpha/2 - g'XB^\mu$, where g and g' are the $SU(4)_\mathrm{L}$ and $U(1)_X$ gauge coupling constants respectively, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm} = g^2(v^2 + v'^2)/2$, $M^2_{K^\pm} = g^2(v'^2 + V^2)/2$, $M^2_{X^\pm} = g^2(v^2 + V'^2)/2$, $M^2_{Y^\pm} = g^2(V^2 + V'^2)/2$, $M^2_{K^0(\bar{K}^0)} = g^2(v^2 + V^2)/2$, and $M^2_{X^0(\bar{X}^0)} = g^2(v'^2 + V'^2)/2$. Since $W^\pm$ does not mix with the other charged bosons we have that $\sqrt{v^2 + v'^2} \approx 174$ GeV as mentioned in the previous section.

For the four neutral gauge bosons we get mass terms of the form

$$
\begin{aligned}
M = \frac{g^2}{2}\Bigg\{ &V^2\left(\frac{g'B^\mu}{g} - \frac{2A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}}\right)^2 \\
&+ V'^2\left(\frac{g'B^\mu}{g} + \frac{3A^\mu_{15}}{\sqrt{6}}\right)^2 \\
&+ v'^2\left(A^\mu_3 + \frac{A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}} - \frac{g'B^\mu}{g}\right)^2 \\
&+ v^2\left(\frac{g'B^\mu}{g} - A^\mu_3 + \frac{A^\mu_8}{\sqrt{3}} + \frac{A^\mu_{15}}{\sqrt{6}}\right)^2\Bigg\}.
\end{aligned}
$$

M is a 4×4 matrix with a zero eigenvalue corresponding to the photon. Once the photon field has been identified, there remains a 3×3 mass matrix for three neutral gauge bosons, Z^μ, Z'^μ and Z''^μ. Since we are interested now in the low energy phenomenology of our model, we can choose $V = V'$ in order to simplify matters. Also, the mixing between the three neutral gauge bosons can be further simplified by choosing $v' = v$. For this particular case the field $Z''^\mu = 2A_8^\mu/\sqrt{6}+A_{15}^\mu/\sqrt{3}$ decouples from the other two and acquires a squared mass $(g^2/2)(V^2+v^2)$. By diagonalizing the remaining 2×2 mass matrix we get two other physical neutral gauge bosons, which are defined through the mixing angle θ between Z_μ, Z'_μ:

$$Z_1^\mu = Z_\mu \cos\theta + Z'_\mu \sin\theta\ ,$$
$$Z_2^\mu = -Z_\mu \sin\theta + Z'_\mu \cos\theta,$$

where

$$\tan(2\theta) = \frac{S_W^2\sqrt{C_{2W}}}{(1+S_W^2)^2 + \frac{V^2}{v^2}C_W^4 - 2}. \qquad (2)$$

$S_W = g'/\sqrt{2g'^2+g^2}$ and C_W are the sine and cosine of the electroweak mixing angle, respectively, and $C_{2W} = C_W^2 - S_W^2$.

The photon field A^μ and the fields Z_μ and Z'_μ are given by

$$\begin{aligned}
A^\mu &= S_W A_3^\mu \\
&\quad + C_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2)^{1/2}B^\mu\right]\ , \\
Z^\mu &= C_W A_3^\mu \\
&\quad - S_W\left[\frac{T_W}{\sqrt{3}}\left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2)^{1/2}B^\mu\right]\ , \\
Z'^\mu &= \frac{1}{\sqrt{3}}(1-T_W^2)^{1/2}\left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}}\right) - T_W B^\mu. \qquad (3)
\end{aligned}$$

We can also identify the Y hypercharge associated with the SM abelian gauge boson as

$$Y^\mu = \frac{T_W}{\sqrt{3}}\left(A_8^\mu - 2\frac{A_{15}^\mu}{\sqrt{2}}\right) + (1-T_W^2)^{1/2}B^\mu. \qquad (4)$$

4.1 Charged currents

The Hamiltonian for the charged currents in the model is given by

$$\begin{aligned}
H^{\rm CC} &= \frac{g}{\sqrt{2}} \\
&\times\Bigg\{ W_\mu^+\left[\left(\sum_{j=2}^{3}\bar u_{a\rm L}\gamma^\mu d_{a\rm L}\right) - \bar u_{1\rm L}\gamma^\mu d_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar\nu_{e\alpha\rm L}\gamma^\mu e_{\alpha\rm L}^-\right)\right] \\
&+K_\mu^+\left[\left(\sum_{j=2}^{3}\bar u_{a\rm L}\gamma^\mu D_{a\rm L}\right) - \bar U_{1\rm L}\gamma^\mu d_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar N_{\alpha\rm L}^0\gamma^\mu e_{\alpha\rm L}^-\right)\right] \\
&+X_\mu^+\left[\left(\sum_{j=2}^{3}\bar U_{a\rm L}\gamma^\mu d_{a\rm L}\right) - \bar u_{1\rm L}\gamma^\mu D_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar\nu_{e\alpha\rm L}\gamma^\mu E_{\alpha\rm L}^-\right)\right] \\
&+Y_\mu^+\left[\left(\sum_{j=2}^{3}\bar U_{a\rm L}\gamma^\mu D_{a\rm L}\right) - \bar U_{1\rm L}\gamma^\mu D_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar N_{\alpha\rm L}^0\gamma^\mu E_{\alpha\rm L}^-\right)\right] \\
&+K_\mu^0\left[\left(\sum_{j=2}^{3}\bar d_{a\rm L}\gamma^\mu D_{a\rm L}\right) - \bar U_{1\rm L}\gamma^\mu u_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar N_{\alpha\rm L}^0\gamma^\mu \nu_{e\alpha\rm L}\right)\right] \\
&+X_\mu^0\left[\left(\sum_{j=2}^{3}\bar u_{a\rm L}\gamma^\mu U_{a\rm L}\right) - \bar D_{1\rm L}\gamma^\mu d_{1\rm L} - \left(\sum_{\alpha=1}^{3}\bar E_{\alpha\rm L}^-\gamma^\mu e_{\alpha\rm L}^-\right)\right]\Bigg\} \\
&+\text{h.c.}
\end{aligned}$$

4.2 Neutral currents

The neutral currents $J_\mu(\text{EM})$, $J_\mu(Z)$, $J_\mu(Z')$, and $J_\mu(Z'')$ associated with the Hamiltonian

$$\begin{aligned}
H^0 &= eA^\mu J_\mu(\text{EM}) + (g/C_W)Z^\mu J_\mu(Z) \\
&\quad + (g')Z'^\mu J_\mu(Z') + (g/(2\sqrt{2}))Z''^\mu J_\mu(Z''),
\end{aligned}$$

are

$$\begin{aligned}
&J_\mu(\text{EM}) \\
&= \frac{2}{3}\left[\sum_{j=2}^{3}(\bar u_a\gamma_\mu u_a + \bar U_a\gamma_\mu U_a) + \bar u_1\gamma_\mu u_1 + \bar U_1\gamma_\mu U_1\right] \\
&\quad -\frac{1}{3}\left[\sum_{j=2}^{3}(\bar d_a\gamma_\mu d_a + \bar D_a\gamma_\mu D_a) + \bar d_1\gamma_\mu d_1 + \bar D_1\gamma_\mu D_1\right] \\
&\quad -\sum_{\alpha=1}^{3}\bar e_\alpha^-\gamma_\mu e_\alpha^- - \sum_{\alpha=1}^{3}\bar E_\alpha^-\gamma_\mu E_\alpha^- \\
&= \sum_f q_f \bar f\gamma_\mu f, \\
&J_\mu(Z) = J_{\mu,\rm L}(Z) - S_W^2 J_\mu(\text{EM}), \\
&J_\mu(Z') = J_{\mu,\rm L}(Z') - T_W J_\mu(\text{EM}), \\
&J_\mu(Z'') \\
&= \sum_{a=2}^{3}(\bar u_{a\rm L}\gamma_\mu u_{a\rm L} + \bar d_{a\rm L}\gamma_\mu d_{a\rm L} - \bar D_{a\rm L}\gamma_\mu D_{a\rm L} - \bar U_{a\rm L}\gamma_\mu U_{a\rm L}) \\
&\quad -\bar d_{1\rm L}\gamma_\mu d_{1\rm L} - \bar u_{1\rm L}\gamma_\mu u_{1\rm L} + \bar U_{1\rm L}\gamma_\mu U_{1\rm L} + \bar D_{1\rm L}\gamma_\mu D_{1\rm L} \\
&\quad +\sum_{\alpha=1}^{3}(-\bar e_{\alpha\rm L}^-\gamma_\mu e_{\alpha\rm L}^- - \bar\nu_{e\alpha\rm L}\gamma_\mu\nu_{e\alpha\rm L} \\
&\quad +\bar N_{\alpha\rm L}^0\gamma_\mu N_{\alpha\rm L}^0 + \bar E_{\alpha\rm L}^-\gamma_\mu E_{\alpha\rm L}^-), \qquad (5)
\end{aligned}$$

where $e = gS_W = g'C_W\sqrt{1-T_W^2} > 0$ is the electric charge, q_f is the electric charge of the fermion f in units of e, and $J_\mu(\text{EM})$ is the electromagnetic current. Note from

$J_\mu(Z'')$ that, notwithstanding the extra neutral gauge boson, Z''_μ does not mix with Z_μ or Z'_μ (for the particular case $V = V'$ and $v = v'$); it still couples to ordinary fermions.

The left-handed currents are

$$\begin{aligned} J_{\mu,\rm L}(Z) &= \frac{1}{2}\Bigg[\sum_{j=2}^{3}(\bar{u}_{a\rm L}\gamma_\mu u_{a\rm L} - \bar{d}_{a\rm L}\gamma_\mu d_{a\rm L}) \\ &\quad -(\bar{d}_{1\rm L}\gamma_\mu d_{1\rm L} - \bar{u}_{1\rm L}\gamma_\mu u_{1\rm L}) \\ &\quad -\sum_{\alpha=1}^{3}(\bar{e}^-_{\alpha\rm L}\gamma_\mu e^-_{\alpha\rm L} - \bar{\nu}_{e\alpha\rm L}\gamma_\mu \nu_{e\alpha\rm L})\Bigg] \\ &= \sum_f T_{4f}\bar{f}_{\rm L}\gamma_\mu f_{\rm L}, \\ J_{\mu,\rm L}(Z') &= (2T_{\rm W})^{-1}\Bigg\{\sum_{j=2}^{3}\big[T_{\rm W}^2(\bar{u}_{a\rm L}\gamma_\mu u_{a\rm L} - \bar{d}_{a\rm L}\gamma_\mu d_{a\rm L}) \\ &\quad -\bar{D}_{a\rm L}\gamma_\mu D_{a\rm L} + \bar{U}_{a\rm L}\gamma_\mu U_{a\rm L}\big] \\ &\quad -T_{\rm W}^2(\bar{d}_{1\rm L}\gamma_\mu d_{1\rm L} - \bar{u}_{1\rm L}\gamma_\mu u_{1\rm L}) \\ &\quad +\bar{U}_{1\rm L}\gamma_\mu U_{1\rm L} - \bar{D}_{1\rm L}\gamma_\mu D_{1\rm L} \\ &\quad +\sum_{\alpha=1}^{3}\big[-T_{\rm W}^2(\bar{e}^-_{\alpha\rm L}\gamma_\mu e^-_{\alpha\rm L} - \bar{\nu}_{\alpha\rm L}\gamma_\mu \nu_{\alpha\rm L}) \\ &\quad +\bar{N}^0_{\alpha\rm L}\gamma_\mu N^0_{\alpha\rm L} - \bar{E}^-_{\alpha\rm L}\gamma_\mu E^-_{\alpha\rm L}\big]\Bigg\} \\ &= \sum_f T'_{4f}\bar{f}_{\rm L}\gamma_\mu f_{\rm L}, \end{aligned} \tag{6}$$

where $T_{4f} = \mathrm{Dg}(1/2, -1/2, 0, 0)$ is the third component of the weak isospin and $T'_{4f} = (1/2T_{\rm W})\mathrm{Dg}(T_{\rm W}^2, -T_{\rm W}^2, -1, 1) = T_{\rm W}\lambda_3/2 + (1/T_{\rm W})(\lambda_8/(2\sqrt{3}) - \lambda_{15}/\sqrt{6})$ is a convenient 4×4 diagonal matrix, acting both of them on the representation 4 of $SU(4)_{\rm L}$. Notice that $J_\mu(Z)$ is just the generalization of the neutral current present in the SM. This allows us to identify Z_μ as the neutral gauge boson of the SM, which is consistent with (3) and (4).

The couplings of the mass eigenstates Z_1^μ and Z_2^μ are given by

$$\begin{aligned} H^{\rm NC} &= \frac{g}{2C_{\rm W}}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu\,[a_{i\rm L}(f)(1-\gamma_5) \\ &\quad + a_{i\rm R}(f)(1+\gamma_5)]\,f\} \\ &= \frac{g}{2C_{\rm W}}\sum_{i=1}^{2} Z_i^\mu \sum_f \{\bar{f}\gamma_\mu\,[g(f)_{iV} - g(f)_{iA}\gamma_5]\,f\}, \end{aligned}$$

where

$$\begin{aligned} a_{1\rm L}(f) &= \cos\theta(T_{4f} - q_f S_{\rm W}^2) \\ &\quad + \frac{g'\sin\theta C_{\rm W}}{g}(T'_{4f} - q_f T_{\rm W}), \\ a_{1\rm R}(f) &= -q_f S_{\rm W}\left(\cos\theta S_{\rm W} + \frac{g'\sin\theta}{g}\right), \\ a_{2\rm L}(f) &= -\sin\theta(T_{4f} - q_f S_{\rm W}^2) \\ &\quad + \frac{g'\cos\theta C_{\rm W}}{g}(T'_{4f} - q_f T_{\rm W}), \\ a_{2\rm R}(f) &= q_f S_{\rm W}\left(\sin\theta S_{\rm W} - \frac{g'\cos\theta}{g}\right), \end{aligned} \tag{7}$$

and

$$\begin{aligned} g(f)_{1V} &= \cos\theta(T_{4f} - 2S_{\rm W}^2 q_f) \\ &\quad + \frac{g'\sin\theta}{g}(T'_{4f}C_{\rm W} - 2q_f S_{\rm W}), \\ g(f)_{2V} &= -\sin\theta(T_{4f} - 2S_{\rm W}^2 q_f) \\ &\quad + \frac{g'\cos\theta}{g}(T'_{4f}C_{\rm W} - 2q_f S_{\rm W}), \\ g(f)_{1A} &= \cos\theta T_{4f} + \frac{g'\sin\theta}{g}T'_{4f}C_{\rm W}, \\ g(f)_{2A} &= -\sin\theta T_{4f} + \frac{g'\cos\theta}{g}T'_{4f}C_{\rm W}. \end{aligned} \tag{8}$$

The values of g_{iV}, g_{iA} with $i = 1, 2$ are listed in Tables 2 and 3.

Table 2. The $Z_1^\mu \longrightarrow \bar{f}f$ couplings

f	$g(f)_{1V}$	$g(f)_{1A}$
$u_{1,2,3}$	$\cos\theta\left(\frac{1}{2} - \frac{4S_{\rm W}^2}{3}\right) - \frac{5\sin\theta}{6(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$d_{1,2,3}$	$\left(-\frac{1}{2} + \frac{2S_{\rm W}^2}{3}\right)\cos\theta + \frac{\sin\theta}{6(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$-\frac{1}{2}\cos\theta - \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$D_{1,2,3}$	$\frac{2S_{\rm W}^2}{3}\cos\theta + \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}\left(\frac{7S_{\rm W}^2}{3} - 1\right)$	$-\frac{\sin\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$U_{1,2,3}$	$-\frac{4S_{\rm W}^2}{3}\cos\theta - \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}\left(\frac{11S_{\rm W}^2}{3} - 1\right)$	$\frac{\sin\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$e^-_{1,2,3}$	$\cos\theta\left(-\frac{1}{2} + 2S_{\rm W}^2\right) + \frac{5\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$-\frac{\cos\theta}{2} - \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$\nu_{1,2,3}$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$\frac{1}{2}\cos\theta + \frac{\sin\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$N^0_{1,2,3}$	$\frac{\sin\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$	$\frac{\sin\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$E^-_{1,2,3}$	$2S_{\rm W}^2\cos\theta + \frac{\sin\theta}{(C_{2\rm W})^{1/2}}\left(2 - \frac{5}{2}C_{\rm W}^2\right)$	$-\frac{\sin\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$

Table 3. The $Z_2^\mu \longrightarrow \bar{f}f$ couplings

f	$g(f)_{2V}$	$g(f)_{2A}$
$u_{1,2,3}$	$-\sin\theta\left(\frac{1}{2}-\frac{4S_{\rm W}^2}{3}\right)-\frac{5\cos\theta}{6(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$d_{1,2,3}$	$\left(\frac{1}{2}-\frac{2S_{\rm W}^2}{3}\right)\sin\theta+\frac{\cos\theta}{6(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$\frac{1}{2}\sin\theta-\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$D_{1,2,3}$	$-\frac{2S_{\rm W}^2}{3}\sin\theta+\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}\left(\frac{7S_{\rm W}^2}{3}-1\right)$	$-\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$U_{1,2,3}$	$\frac{4S_{\rm W}^2}{3}\sin\theta-\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}\left(\frac{11S_{\rm W}^2}{3}-1\right)$	$\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$e^-_{1,2,3}$	$\sin\theta\left(\frac{1}{2}-2S_{\rm W}^2\right)+\frac{5\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$\frac{\sin\theta}{2}-\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$\nu_{1,2,3}$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$	$-\frac{1}{2}\sin\theta+\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}S_{\rm W}^2$
$N^0_{1,2,3}$	$\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$	$\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$
$E^-_{1,2,3}$	$-2S_{\rm W}^2\sin\theta+\frac{\cos\theta}{(C_{2\rm W})^{1/2}}\left(2-\frac{5}{2}C_{\rm W}^2\right)$	$-\frac{\cos\theta}{2(C_{2\rm W})^{1/2}}C_{\rm W}^2$

As we can see, in the limit $\theta = 0$ the couplings of Z_1^μ to the ordinary leptons and quarks are the same as in the SM; due to this we can test the new physics beyond the SM predicted by this particular model.

5 Fermion masses

The Higgs scalars introduced in Sect. 3 break the symmetry in an appropriate way. Now, in order to generate both a simple mass splitting between ordinary and exotic fermions and a consistent mass spectrum, we introduce an anomaly-free discrete Z_2 symmetry [6], with the following assignments of Z_2 charge q:

$$\begin{aligned} q(Q_{a\rm L}, u^c_{a\rm L}, d^c_{a\rm L}, L_{a\rm L}, e^c_{a\rm L}, \phi_1, \phi_3) &= 0,\\ q(U^c_{a\rm L}, D^c_{a\rm L}, E^c_{a\rm L}, \phi_2, \phi_4) &= 1. \end{aligned} \tag{9}$$

Notice that ordinary fermions are not affected by this discrete symmetry.

The gauge invariance and the Z_2 symmetry allow for the following Yukawa lagrangians.
(1) For quarks:

$$\begin{aligned} \mathcal{L}_Y^Q = \sum_{j=2}^{3} Q^{\rm T}_{a\rm L} C \Bigg\{ &\phi_3^* \sum_{\alpha=1}^{3} h^u_{j\alpha} u^c_{\alpha\rm L} + \phi_4^* \sum_{\alpha=1}^{3} h^U_{j\alpha} U^c_{\alpha\rm L} \\ &+ \phi_1 \sum_{\alpha=1}^{3} h^d_{j\alpha} d^c_{\alpha\rm L} + \phi_2 \sum_{\alpha=1}^{3} h^D_{j\alpha} D^c_{\alpha\rm L} \Bigg\} \\ + Q^{\rm T}_{1\rm L} C \Bigg\{ &\phi_1^* \sum_{\alpha=1}^{3} h^u_{1\alpha} u^c_{\alpha\rm L} + \phi_2^* \sum_{\alpha=1}^{3} h^U_{1\alpha} U^c_{\alpha\rm L} \\ &+ \phi_3 \sum_{\alpha=1}^{3} h^d_{1\alpha} d^c_{\alpha\rm L} + \phi_4 \sum_{\alpha=1}^{3} h^D_{1\alpha} D^c_{\alpha\rm L} \Bigg\} + \text{h.c.}, \end{aligned}$$

where the $h's$ are Yukawa couplings and C is the charge conjugate operator.
(2) For charged leptons:

$$\mathcal{L}_Y^l = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} L^{\rm T}_{\alpha\rm L} C \left\{ \phi_3 h^e_{\alpha\beta} e^+_{\beta\rm L} + \phi_4 h^E_{\alpha\beta} E^+_{\beta\rm L} \right\} + \text{h.c.}$$

The lagrangian $\mathcal{L}_Y^Q$ produces for up- and down-type quarks, in the basis (u_1, u_2, u_3, U_1, U_2, U_3) and (d_1, d_2, d_3, D_1, D_2, D_3) respectively, 6×6 block diagonal mass matrices of the form

$$M_{uU} = \begin{pmatrix} M_{u(3\times3)} & 0 \\ 0 & M_{U(3\times3)} \end{pmatrix},$$

where

$$M_u = \begin{pmatrix} h^u_{11}v & h^u_{21}v' & h^u_{31}v' \\ h^u_{12}v & h^u_{22}v' & h^u_{32}v' \\ h^u_{13}v & h^u_{23}v' & h^u_{33}v' \end{pmatrix},$$

$$M_U = \begin{pmatrix} h^U_{11}V & h^U_{21}V' & h^U_{31}V' \\ h^U_{12}V & h^U_{22}V' & h^U_{32}V' \\ h^U_{13}V & h^U_{23}V' & h^U_{33}V' \end{pmatrix},$$

and

$$M_{dD} = \begin{pmatrix} M_{d(3\times3)} & 0 \\ 0 & M_{D(3\times3)} \end{pmatrix},$$

where

$$M_d = \begin{pmatrix} h^d_{11}v' & h^d_{21}v & h^d_{31}v \\ h^d_{12}v' & h^d_{22}v & h^d_{32}v \\ h^d_{13}v' & h^d_{23}v & h^d_{33}v \end{pmatrix},$$

$$M_D = \begin{pmatrix} h^D_{11}V' & h^D_{21}V & h^D_{31}V \\ h^D_{12}V' & h^D_{22}V & h^D_{32}V \\ h^D_{13}V' & h^D_{23}V & h^D_{33}V \end{pmatrix}.$$

For the charged leptons the lagrangian $\mathcal{L}_Y^l$, in the basis $(e_1, e_2, e_3, E_1, E_2, E_3)$, also produces a block diagonal mass matrix

$$M_{eE} = \begin{pmatrix} M_{e(3\times3)} & 0 \\ 0 & M_{E(3\times3)} \end{pmatrix},$$

where the entries in the submatrices are given by

$$M_{e,\alpha\beta} = h^e_{\alpha\beta}v' \qquad \text{and} \qquad M_{E,\alpha\beta} = h^E_{\alpha\beta}V'.$$

The former mass matrices exhibit the mass splitting between ordinary and exotic charged fermions and show that all the charged fermions in the model acquire masses at the tree level. Clearly, by a judicious tuning of the Yukawa couplings and of the mass scales v and v', a consistent mass spectrum in the ordinary charged sector can be obtained. In the exotic charged sector all the particles acquire masses at the scale $V \sim V' \gg 174\,\text{GeV}$. Note that in the low energy limit our model corresponds to a Type III two Higgs doublet model [7] in which both doublets couple to the same type of fermions, with the quark and lepton couplings treated asymmetrically.

The neutral leptons remain massless as far as we use only the original fields introduced in Sect. 2. But as mentioned earlier, we may introduce one or more Weyl singlet states $N^0_{L,b}$, $b = 1, 2, \ldots$, which may implement the appropriate neutrino oscillations [8].

6 Constraints on the $(Z^\mu$–$Z'^\mu)$ mixing angle and the Z_2^μ mass

To bound $\sin\theta$ and M_{Z_2} we use parameters measured at the Z pole from CERN e^+e^- collider (LEP), SLAC Linear Collider (SLC), and atomic parity violation constraints which are given in Table 4.

The expression for the partial decay width for $Z_1^\mu \to f\bar{f}$ is

$$\begin{aligned}&\Gamma(Z_1^\mu \to f\bar{f})\\ &= \frac{N_C G_F M_{Z_1}^3}{6\pi\sqrt{2}}\rho\left\{\frac{3\beta-\beta^3}{2}\left[g(f)_{1V}\right]^2 + \beta^3\left[g(f)_{1A}\right]^2\right\}\\ &\quad\times(1+\delta_f)R_{\text{EW}}R_{\text{QCD}},\end{aligned} \tag{10}$$

where f is an ordinary SM fermion, Z_1^μ is the physical gauge boson observed at LEP, $N_C = 1$ for leptons while for quarks $N_C = 3(1 + \alpha_s/\pi + 1.405\alpha_s^2/\pi^2 - 12.77\alpha_s^3/\pi^3)$, where the 3 is due to color and the factor in parentheses represents the universal part of the QCD corrections for massless quarks (for fermion mass effects and further QCD corrections which are different for vector and axial-vector partial widths, see [9]); R_{EW} is for the electroweak corrections which include the leading order QED corrections given by $R_{\text{QED}} = 1 + 3\alpha/(4\pi)$. R_{QCD} denotes further QCD corrections (for a comprehensive review, see [10] and references therein), and $\beta = \sqrt{1 - 4m_f^2/M_{Z_1}^2}$ is a kinematic factor which can be taken equal to 1 for all the SM fermions except for the bottom quark. The factor δ_f contains the one loop vertex contribution which is negligible for all fermion fields except for the bottom quark, for which the contribution coming from the top quark at the one loop vertex radiative correction is parametrized as $\delta_b \approx 10^{-2}\left[-m_t^2/(2M_{Z_1}^2) + 1/5\right]$ [11]. The ρ parameter can be expanded as $\rho = 1 + \delta\rho_0 + \delta\rho_V$ where the oblique correction $\delta\rho_0$ is given by $\delta\rho_0 \approx 3G_F m_t^2/(8\pi^2\sqrt{2})$, and $\delta\rho_V$ is the tree level contribution due to the $(Z_\mu$–$Z'_\mu)$ mixing which can be parametrized as $\delta\rho_V \approx (M_{Z_2}^2/M_{Z_1}^2 - 1)\sin^2\theta$. Finally, $g(f)_{1V}$ and $g(f)_{1A}$ are the coupling constants of the physical Z_1^μ field with ordinary fermions which are listed in Table 2.

In what follows we are going to use the experimental values [12] $M_{Z_1} = 91.188\,\text{GeV}$, $m_t = 174.3\,\text{GeV}$, $\alpha_s(m_Z) = 0.1192$, $\alpha(m_Z)^{-1} = 127.938$, and $\sin^2\theta_W = 0.2333$. The experimental values are introduced using the definitions $R_\eta \equiv \Gamma(\eta\eta)/\Gamma(\text{hadrons})$ for $\eta = e, \mu, \tau, b, c$.

As a first result notice from Table 2, that our model predicts $R_e = R_\mu = R_\tau$, in agreement with the experimental results in Table 4.

The effective weak charge in atomic parity violation, Q_W, can be expressed as a function of the number of protons (Z) and the number of neutrons (N) in the atomic nucleus in the form

$$Q_W = -2\left[(2Z+N)c_{1u} + (Z+2N)c_{1d}\right], \tag{11}$$

where $c_{1q} = 2g(e)_{1A}g(q)_{1V}$. The theoretical value for Q_W for the cesium atom is given by [13] $Q_W(^{133}_{55}\text{Cs}) = -73.09 \pm 0.04 + \Delta Q_W$, where the contribution of new physics is included in ΔQ_W, which can be written as [14]

$$\Delta Q_W = \left[\left(1 + 4\frac{S_W^4}{1 - 2S_W^2}\right)Z - N\right]\delta\rho_V + \Delta Q'_W. \tag{12}$$

The term $\Delta Q'_W$ is model dependent and it can be obtained for our model by using $g(e)_{iA}$ and $g(q)_{iV}$, $i = 1, 2$, from Tables 2 and 3. The value we obtain is

$$\begin{aligned}\Delta Q'_W &= (3.75Z + 2.56N)\sin\theta\\ &\quad+ (1.22Z + 0.41N)\frac{M_{Z_1}^2}{M_{Z_2}^2}\ .\end{aligned} \tag{13}$$

The discrepancy between the SM and the experimental data for ΔQ_W is given by [15]

$$\Delta Q_W = Q_W^{\text{exp}} - Q_W^{\text{SM}} = 1.03 \pm 0.44, \tag{14}$$

which is 2.3 σ away from the SM predictions.

Introducing the expressions for Z pole observables in (10), with ΔQ_W in terms of new physics in (12) and using experimental data from LEP, SLC and atomic parity violation (see Table 4), we do a χ^2 fit and we find the best

Table 4. Experimental data and SM values for the parameters

	Experimental results	SM
Γ_Z (GeV)	2.4952 ± 0.0023	2.4966 ± 0.0016
$\Gamma(\text{had})$ (GeV)	1.7444 ± 0.0020	1.7429 ± 0.0015
$\Gamma(l^+l^-)$ (MeV)	83.984 ± 0.086	84.019 ± 0.027
R_e	20.804 ± 0.050	20.744 ± 0.018
R_μ	20.785 ± 0.033	20.744 ± 0.018
R_τ	20.764 ± 0.045	20.790 ± 0.018
R_b	0.21664 ± 0.00068	0.21569 ± 0.00016
R_c	0.1729 ± 0.0032	0.17230 ± 0.00007
Q_W^{Cs}	$-72.65 \pm 0.28 \pm 0.34$	-73.10 ± 0.03
M_{Z_1} (GeV)	91.1872 ± 0.0021	91.1870 ± 0.0021

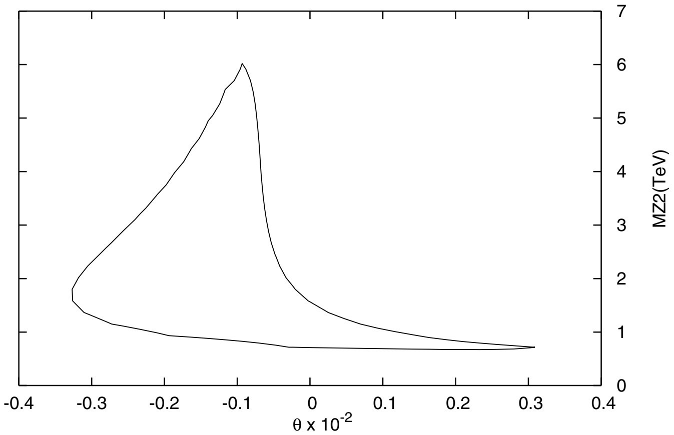

Fig. 1. Contour plot displaying the allowed region for θ versus M_{Z_2} at 95% C.L.

allowed region in the (θ–M_{Z_2}) plane at 95% confidence level (C.L.). In Fig. 1 we display this region, which gives us the constraints

$$-0.0032 \leq \theta \leq 0.0031, \quad 0.67\,\text{TeV} \leq M_{Z_2} \leq 6.1\,\text{TeV}. \tag{15}$$

As we can see, the mass of the new neutral gauge boson is compatible with the bound obtained in $p\bar{p}$ collisions at the Fermilab Tevatron [16].

7 Conclusions

We have presented an anomaly-free model based on the local gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$, which does not contain exotic electric charges. This last constraint fixes the values $b = 1$ and $c = -2$ for the parameters in the electric charge generator in (1).

We break the gauge symmetry down to $SU(3)_c \otimes U(1)_Q$ in an appropriate way by using four different Higgs scalars ϕ_i, $i = 1, 2, 3, 4$, which set two different mass scales: $V \sim V' >> \sqrt{v^2 + v'^2} \simeq 174\,\text{GeV}$, with $v \sim v'$. By introducing an anomaly-free discrete Z_2 symmetry we also obtain a simple mass splitting between exotic and ordinary fermions, and a consistent mass spectrum both in the quark and in the lepton sector. Notice also the consistence of our model in the charged lepton sector where it predicts the correct ratios R_η, $\eta = e, \mu, \tau$, in the Z decays. This is a characteristic feature of the two classes of three-family models introduced in [3].

By using experimental results we obtain a lowest bound of $0.67\,\text{TeV} \leq M_{Z_2}$ for the mass of an extra neutral gauge boson Z_2, and we find the bound of the mixing angle θ between the SM neutral current and the Z_2 one to be $-0.0032 < \theta < 0.0031$.

When we compare the numerical results presented in the previuos section with the results presented in [3], we find that the mixing angle θ is of the same order of magnitude ($\sim 10^{-3}$), but for the model considered here the mass associated with the new neutral current has smaller lower and upper bounds, with the lower bound just below the TeV scale, which allows for a possible signal at the Fermilab Tevatron.

For our analysis we have chosen just one of the two possible three-family models without exotic electric charges, characterized by the parameters $b = -c/2 = 1$ in the electric charge operator [3]. We believe that the low energy phenomenology for the other model must produce results similar to the ones presented in this paper.

Acknowledgements. We acknowledge partial financial support from Universidad Nacional de Colombia-Sede Medellín, and from Universidad de Antioquia. We also thank Diego A. Gutiérrez for helping us with the numerical analysis presented in Sect. 6.

References

1. For an excellent compendium of the SM see: J.F. Donoghue, E. Golowich, B. Holstein, Dynamics of the standard model (Cambridge University Press, Cambridge, England 1992)
2. M.B. Voloshin, Sov. J. Nucl. Phys. **48**, 512 (1988); F. Pisano, T.A. Tran, ICTP preprint IC/93/200, Proceedings of The XIV Encontro National de Física de Partículas e Campos, Caxambu (1993); V. Pleitez, Report No. IFT-P.010/93, hep-ph/9302287; R. Foot, H.N. Long, T.A. Tran, Phys. Rev. D **50**, R34 (1994); F. Pisano, V. Pleitez, Phys. Rev. D **51**, 3865 (1995); I. Cotaescu, Int. J. Mod. Phys. A **12**, 1483 (1997)
3. W.A. Ponce, D.A. Gutiérrez, L.A. Sánchez, Phys. Rev. D **69**, 055007 (2004)
4. D.E. Kaplan, M. Schmaltz, JHEP **0310**, 039 (2003); O.C.W. Kong, hep-ph/0307250, NCU-HEP-K009; hep-ph/0308148
5. W.A. Ponce, J.B. Flórez, L.A. Sánchez, Int. J. Mod. Phys. A **17**, 643 (2002); W.A. Ponce, Y. Giraldo, L.A. Sánchez, Systematic study of 3-3-1 models, in Proceedings of the VIII Mexican Workshop of Particles and Fields, Zacatecas, Mexico, 2001, edited by J.L. Díaz-Cruz et al. (AIP Conf. Proc. Vol. 623, New York, 2002), pp. 341–346 [hep-ph/0201133]
6. L.M. Krauss, F. Wilczek, Phys. Rev. Lett. **62**, 1221 (1989); L.E. Ibañez, G.G. Ross, Phys. Lett. B **260**, 291 (1991)
7. W.S. Hou, Phys. Lett. B **296**, 179 (1992); D. Cahng, W.S. Hou, W.Y. Keung, Phys. Rev. D **48**, 217 (1993); S. Nie, M. Sher, Phys. Rev. D **58**, 097701 (1998)
8. For recent reviews see: J.W.F. Valle, Neutrino masses twenty-five years later, hep-ph/0307192; V. Barger, D. Marfalia, K. Whishnaut, Int. J. Mod. Phys. E **12**, 569 (2003)
9. K.G. Chetyrkin, J.H. Kün, Phys. Lett. B **406**, 102 (1997)
10. A. Leike, Phys. Rep. **317**, 143 (1999)
11. J. Bernabeu, A. Pich, A. Santamaria, Nucl. Phys. B **363**, 326 (1991)
12. Particle Data Group, K. Hagiwara et al., Phys. Rev. D **66**, 01001 (2002)
13. W.J. Marciano, A. Sirlin, Phys. Rev D **29**, 75 (1984); W.J. Marciano, J. Rosner, Phys. Rev. Lett. **65**, 2963 (1990)
14. L. Durkin, P. Langacker, Phys. Lett. B **166**, 436 (1986)
15. R. Casalbuoni, S. De Curtis, D. Dominici, R. Gatto, Phys. Lett. B **460**, 135 (1999); J. Rosner, Phys. Rev. D **61**, 016006 (2000); J. Erler, P. Langacker, Phys. Rev. Lett. **84**, 212 (2000)
16. F. Abe et al., Phys. Rev. Lett. **79**, 2192 (1997)